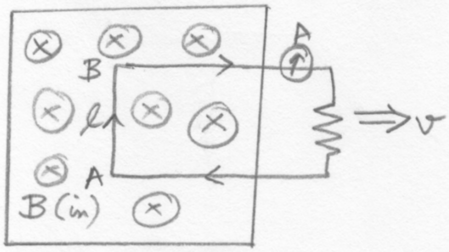
Lecture 21 Electromagnetic Induction

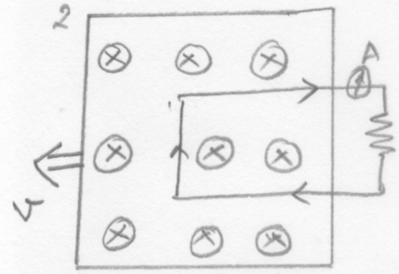
Michael Faraday, 1831

1.



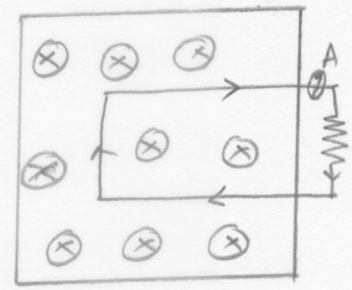
Moving wire

2



Moving magnet

3



$\frac{dB}{dt} \neq 0$

1. Moving wire = Motional EMF

$$\vec{F} = q\vec{v} \times \vec{B} \quad \text{on a single charge}$$

Therefore, work done by charge in going from A to B is:

$$W = \vec{F} \cdot \vec{dr} = F \cdot l = qvBl$$

Now $W = qV_{emf}$; $V_{emf} \equiv \mathcal{E}_{emf}$ (same units of Voltage. Does the same thing as voltage.)

$$V_{emf} = vBl$$

$$= lB\frac{dx}{dt}$$

$$\boxed{V_{emf} = -\frac{d\Phi_{flux}}{dt}}$$

$\mathcal{E} = -\frac{d\Phi}{dt}$ In other textbooks.

$\boxed{I = \frac{V_{emf}}{R}} \rightarrow$ observed

2. By Galilean relativity, $\boxed{V_{emf} = -\frac{d\Phi_{emf}}{dt}}$ as well. "Faraday's Law"

But this must come from changing magnetic field!

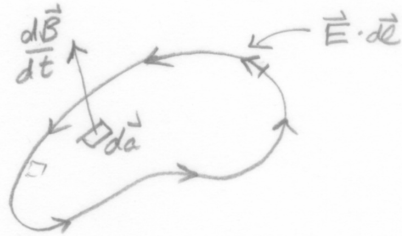
3. If V_{emf} can be generated by changing magnetic field, can we just change it without moving anything? Yes!

We observe that $V_{emf} = -A\frac{dB}{dt}$

area of loop that field cuts

$$V_{emf} = -\text{Area}\,\frac{dB}{dt}$$

$$= \oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$



$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = -\int_S \frac{\partial \vec{B}}{\partial t}\, d\vec{a}$$

$$\therefore \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Fundamental Law "Faraday's Law"

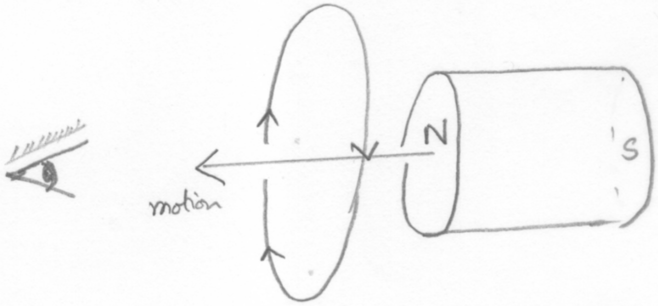
From first principles, there seem to be two different mechanisms for Faradays Law:

① Motional EMF due to motion in a magnetic field.

② Changing magnetic field.

From $V_{emf} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$ can you work out direction?

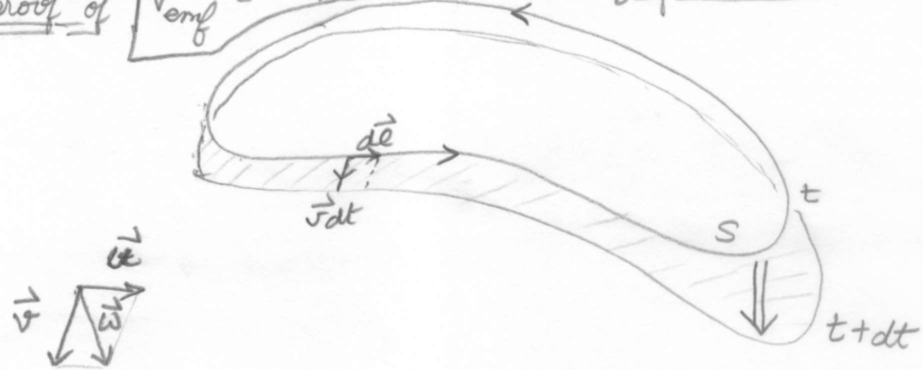
Lenz's Law → nature works to oppose direction of motion that creates the current



Can we also get this from considering motional emf?

Try it yourself!

General proof of $\boxed{V_{emf} = -d\Phi/dt}$ from Lorentz force Law



A loop of wire moves from t to $t+dt$.

$$d\Phi = \Phi(t+dt) - \Phi(t) = \Phi_{ribbon} = \int_{ribbon} \vec{B}\, d\vec{a}.$$

$$d\vec{a} = \vec{v}\,dt \times d\vec{l}$$

$$\therefore \frac{d\Phi}{dt} = \oint \vec{B}\cdot(\vec{v}\times d\vec{l})$$

$$\frac{d\Phi}{dt} = \oint \vec{B}\cdot\left((\vec{u}+\vec{v})\times d\vec{l}\right) \quad \text{because } \vec{u}\times d\vec{l} = 0$$

$$= \oint \vec{B}\cdot(\vec{w}\times d\vec{l})$$

But, $\vec{B}\cdot(\vec{w}\times d\vec{l}) = -(\vec{w}\times\vec{B})\cdot d\vec{l}$

So, $\dfrac{d\Phi}{dt} = -\oint (\vec{w}\times\vec{B})\cdot d\vec{l}$

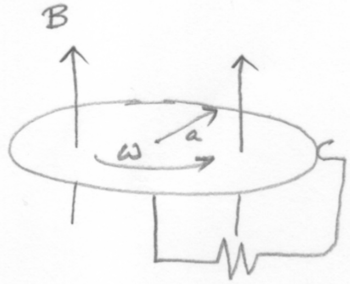
But, $\vec{F} = q(\vec{w}\times\vec{B}) = q\vec{f}_{mag}$

$$\therefore \frac{d\Phi}{dt} = -\oint \vec{f}_{mag}\, d\vec{l}$$

$$\therefore \boxed{V_{emf} = -\frac{d\Phi}{dt}}$$

Example problems

① Use Lorentz force Law



$$f_{mag} = vB$$

$$= \omega r B$$

$$\therefore \; V_{emf} = \int_0^a f_{mag} \cdot dr = B\omega \int_0^a r\,dr$$

$$= \frac{B\omega a^2}{2}$$