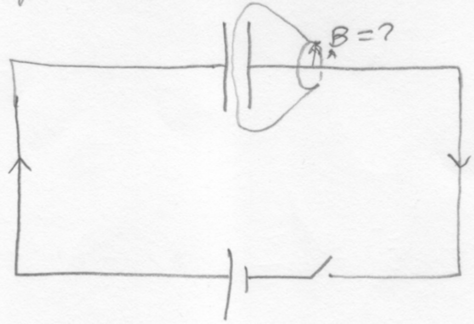
Lecture 22 Displacement Current



We can use Amperes Law in integral form to find B at a distance r from the wire. This cuts a current I. So, $B \cdot 2\pi r = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi r}.$$

But what if we took the surface through the capacitor? There is no current there. This gives a contradiction, a paradox.

Applying divergence on $\vec{\nabla}\times\vec{E}$

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E})}_{0} = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) = \underbrace{-\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B})}_{0}$$

But, applying divergence on $\vec{\nabla}\times\vec{B}$

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})}_{0} = \vec{\nabla}\cdot(\mu_0\vec{J})$$

not necessarily zero!

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$ — Continuity Equation

But, $\rho = \epsilon_0 \vec{\nabla}\cdot\vec{E}$

So, $$\vec{\nabla}\cdot\vec{J} = -\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})$$

So, if we add exactly the $\epsilon_0\frac{\partial\vec{E}}{\partial t}$, we can kill the divergence

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$ — Law of Nature

Then, $$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\vec{\nabla}\cdot\vec{J} + \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\cdot\vec{E})}{\partial t}$$

$$= \mu_0\underbrace{\left[\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t}\right]}_{0}$$