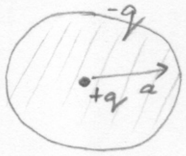
Lecture 7 Dielectrics

Can only partially shield their inside from an external $\vec{E}$ field.

External field acting on an atom.



We had shown before for conductors that $\vec{p} \propto \vec{E}$

For a shell conductor,
$$\vec{p} = 4\pi\epsilon_0 R^3 \vec{E}$$

But in general,
$$\vec{p} = \underbrace{\alpha}_{\text{polarizability}} \vec{E}$$
This is a definition

What is the polarizability of a primitive atom?

Placed in an external electric field $\vec{E}$, we can imagine the nucleus $+q$ has shifted a distance d from the center. Then, the $\vec{E}_e$ return force field it feels is:
$$E_e = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}.$$

This must balance the external field E.

So,
$$E = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3} = \frac{1}{4\pi\epsilon_0}\frac{p}{a^3}$$

So,
$$p = \underbrace{4\pi\epsilon_0 a^3}_{\alpha} E$$

Similar to a shell!

This is a crude model, but within factors of 4 from reality.

According to QM
$$\rho(r) = \frac{q}{\pi a^3} e^{-r/a} \quad a \rightarrow \text{Bohr radius.}$$
What is the polarizability?

In full generality, polarizability is a tensor. Polarization along one axis $\neq$ other axis

$$\vec{p} = \alpha_{\perp} \vec{E}_{\perp} + \alpha_{\parallel} \vec{E}_{\parallel}$$

So
$$p_x = \alpha_{xx}E_x + \alpha_{xy}E_y + \alpha_{xz}E_z$$
$$p_y = \alpha_{yx}E_x + \alpha_{yy}E_y + \alpha_{yz}E_z$$
$$p_z = \alpha_{zx}E_x + \alpha_{zy}E_y + \alpha_{zz}E_z$$

So,
$$p_i = \underbrace{\alpha_{ij}}_{\text{tensor}} E_j$$

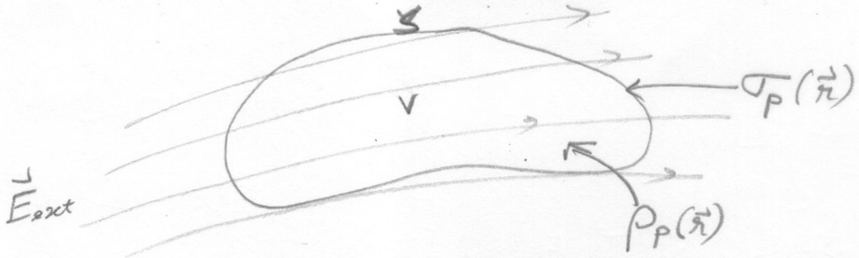
One can always choose a "principal axis" such that all off-diagonal terms = 0.

Electric fields on a dipolar molecule

Will get them to swivel if they are in liquid form.

- What is force of a point charge q a large distance from an atom of polarizability α?

More generally, what happens to a macroscopic body?



When $\vec{E}_{ext} = 0$, $\rho = 0$ in the dielectric

When $\vec{E}_{~~ext~~}$ is applied, $\rho_p(\vec{r})$ is set up such that $\rho_p(\vec{r})\, \underset{\cancel{tot}}{E}(\vec{r})$ experiences equal and opposite force to all internal structural forces.

Total charge density $\rho(\vec{r}) = \rho_{free}(\vec{r}) + \rho_p(\vec{r})$

$\hookrightarrow$ caused the $\vec{E}_{ext}$

The macroscopic charge density is like this:

$$\int_V dV\, \rho_p(\vec{r}) = -\int_S dA\, \sigma_p(\vec{r})$$] this makes it overall neutral.

This would suggest

For a conductor, $\rho_p(\vec{r}) = 0$.

$$\rho_p(\vec{r}) = -\vec{\nabla}\cdot\vec{P}(\vec{r}) \quad ; r \in V$$

So, $$\sigma_p(\vec{r}_s) = \vec{P}(\vec{r}_s)\cdot\hat{\eta}(\vec{r}_s) \quad ; r \in S$$

This $\vec{P}(\vec{r})$ is the polarization.

The macroscopic ~~electric fields~~ produced by surface polarization and bulk polarization.

$\sigma_p(\vec{r})$ $\qquad\qquad$ $\rho_p(\vec{r})$

So, what is the physical meaning of $\vec{P}(\vec{r})$?

Clue to the physical meaning, or underlying microscopic interpretation of, $\vec{P}$ comes from the fact that $\vec{P}(\vec{r})$'s volume integral is equal to the dipole moment of the sample. To see this, look at the x-direction of $\vec{P}(\vec{r})$ and integrate it over the sample.

That is, what is $\int_V P_x(\vec{r})\, dV = ?$

We start with $$\vec{\nabla}\cdot(x\vec{P}) = \underbrace{\vec{P}\cdot\vec{\nabla}x}_{P_x} + x\,\vec{\nabla}\cdot\vec{P}$$

So, $P_x = \vec{\nabla}\cdot(x\vec{P}) - x\vec{\nabla}\cdot\vec{P}$

Therefore,

$$\int_V dV\, P_x = \int_V \vec{\nabla}\cdot(x\vec{P})\, dV - \int_V x\,\vec{\nabla}\cdot\vec{P}\, dV$$

$$= \oint_S x\,\vec{P}\cdot\hat{\eta}\, dS + \int_V x\,\rho_p(\vec{r})\, dV$$

$$= \int_S x\,\sigma_p(r_s)\, dS + \int_V x\,\rho_p(\vec{r})\, dV$$

= dipole moment of the sample. $= \vec{p}$

So,

$$\boxed{\int_V dV\, P(\vec{r}) = \vec{p}}$$